ANALYSIS AND SIMULATION OF POLITICAL PARTIES WITH DIFFERENT STATISTICAL DISTRIBUTIONS

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ABSTRACT
Statistical results of federal deputies' elections by political party were analyzed, observing a normal distribution for the major parties and a heavy tails distribution for the minor parties and/or with disappearing tendencies. The heavy tails distribution that particularly predominated was the log-logistic type. With the alpha shape parameter of the log-logistic distribution minor to two, we found that the small parties tend to disappear. Probabilistic simulations were realized in a model with two parties and with different distributions (normal and log-logistic) and we found that, indeed, the log-logistic distribution party with its alpha parameter minor to two tends to win a few times in an election. Therefore, this produces a loss of supporters over time.

Keywords: simulation, political party, normal distribution, log-logistic distribution

1. INTRODUCTION

Simple models about the cooperative behavior in the social systems were already known by economists and sociologists for many years. What is really surprising is that many of these classic models in sociology were simply reformulated in terms of existing models in statistical mechanics, such as the "Minority Game"(Weidlich 2000), the Axelrod Model of formation of cultural dominions or political coalitions (Schofield and Parks 2000), the economic models based on the Nash local equilibrium concept (Adams and Merrill 1999), among others (Bergstrom 2001). A cooperative behavior in the democratic societies concerns to the elections, which are convincing democratic processes, where the same type of interaction between the voters and external influences (political publicity, campaigns, etc) exists. Even so, the voters do not own as much freedom, since the individual preferences depend on the election of the social networks where the voter is immersed. This is natural in the human being. According to this, a quantitative characterization of the electoral network is made through a study of the vote's distributions. These vote's distributions are obtained from different electoral processes and the simulations of the electoral preferences, which are indispensables steps towards a better understanding and prediction of the underlying electoral dynamics (Martínez and Balankin 2003).

From the scientific point of view of complex system, the result of an electoral process can be considered like an answer of an open system, with many elements that interact with each other, governed by an intern complex dynamic (although unknown). The modeling of the Mexican electoral system belongs to the dynamics of the complex social systems, which has been developed in order to improve the quality in the participation of the democratic processes.

This paper is focused on the statistical behavior of some Mexican political parties and their simulation. Particularly, the results of the elections for Federal Deputies were analyzed in the 1991, 1994, 1997, 2000 and 2003 electoral years.

In Mexico, the Federal Electoral Institute (IFE by its abbreviation in Spanish) is an autonomous public entity in charge of organize the federal elections, that is, the President's, deputies' and senators' elections. Due to this, the IFE is the commissioned public entity to provide electoral statistical information.

1.1. Computational Resources

Netlogo's computer programming language was used for the simulation. This program is focused on the simulation of phenomena in which many individuals appear interacting (e.g. habitual phenomena that occur in nature, society or many fields in mathematics). It is useful to model complex systems that evolve over time. Also, it is used to model thousands of individuals (people, bacteria, insects, organizations, nodes of a graph, etc) that interact with each other and with the environment. It allows to explore the connection between local interactions at an individual level and the macroscopic patterns that emerge of that interactions.

2. STATISTICAL ANALYSIS OF SOME POLITICAL PARTIES

The analysis was made on the elections of Mexico's federal deputies considering the three main parties, during the 1991-2003 electoral years of the 300 districts of the Mexican republic. Statistical data was obtained from the Federal Electoral Institute's data bases (IFE by its abbreviation in Spanish) for the information processing. Then it was exported to Excel and with the

@Risk 4.5 software aid, the votes percentage distribution of each political party of the 300 electoral districts was analyzed. Each political party shows different properties.

Table 1 shows the votes adjustment parameters to the normal and log-logistic distribution, equation 1 and 2 respectively (Law and Kelton 2000), for the National Action Party (PAN by its abbreviation in Spanish).

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} \qquad (1)$$

Where μ is the mean and σ^2 is the variance.

$$f(x) = \frac{\alpha t^{\alpha-1}}{\beta(1+t^{\alpha})^2}; \quad \text{with} \quad t = \frac{x-\gamma}{\beta} \qquad (2)$$

Where γ is the location parameter, β is the scale parameter and α the shape parameter. Table1 shows a normal distribution with growing mean, which indicates a tendency to gain supporters over the years. And a log-logistic distribution with an increasing α parameter, which means a good electoral presence in almost all of the federal entities. Figure 1 shows the normal distribution graphic for PAN in 2003.

Table 1: Mean and Standard Deviation of the Normal and Log-Logistic Distributions for Votes Percentage of PAN

PAN	Normal		Log-Logistic	
	μ	σ	α	β
1991	0.1685	0.1090	0.1617	0.0631
1994	0.2499	0.0932	0.2515	0.0534
1997	0.2636	0.1241	Extremal value	
2000	0.4218	0.1051	0.4296	0.0606
2003	0.3017	0.1172	0.3089	0.0606

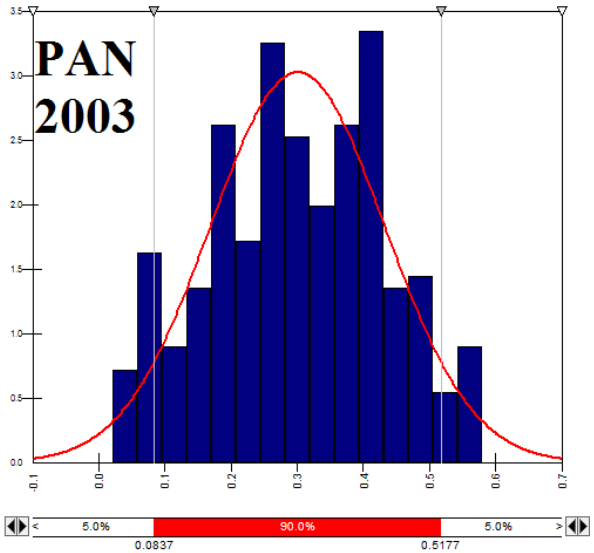

Figure 1: Votes Distribution graphic for PAN in 2003 in Mexico's Electoral Districts

In table 2 is observed that, in opposition to PAN, the Institutional Revolutionary Party (PRI by its abbreviation in Spanish) presents a normal mean which tends to decrease, as the same as the log-logistic α parameter. This means that the party tends to lose supporters over time. Nevertheless, it continues remaining strong like a national party, since the distribution of its supporters continues being normal with an acceptable average.

Table 2: Mean and Standard Deviation of the Normal and Log-Logistic distributions for Votes Percentage of PRI

PRI	Normal		Log-Logistic	
	μ	σ	α	β
1991	0.6210	0.1265	0.6197	0.0762
1994	0.5071	0.0913	0.5030	0.0537
1997	0.4087	0.1085	0.4075	0.0630
2000	0.3910	0.1071	0.3892	0.0633
2003	0.3793	0.1410	0.3903	0.0800

In figure 2 is shown graphically the votes distribution of PRI in 1997. As the same as PAN, data adjust to a normal distribution and with less probability it adjust to a heavy tails distribution.

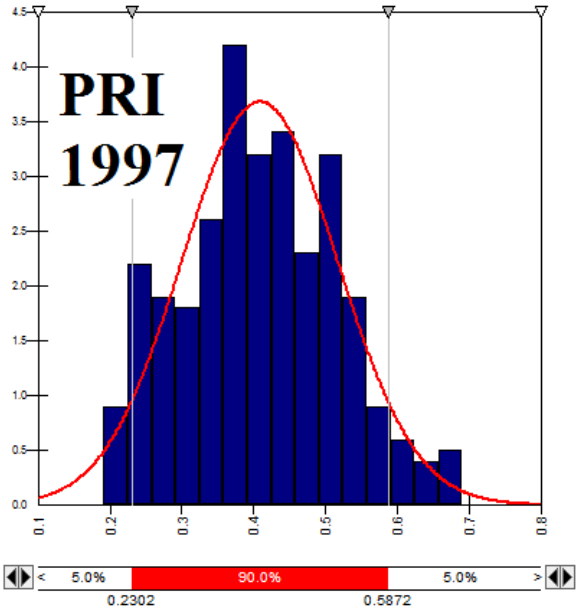

Figure 2: Votes Distribution graphic for PRI in 1997 in Mexico's Electoral Districts

Table 3 shows the votes adjustment parameters to the log-logistic and Pearson distributions of the Democratic Revolution Party (PRD by its abbreviation in Spanish).Unlike PAN and PRI, PRD preserves a log-logistic distribution given by equation 2; it means that there is no a voters' presence in many federal entities and that suggest that the party tends to disappear over the years.

Table 3: Parameters of the Log-Logistic and Pearson Distributions for Votes Percentage of PRD

PRD	Normal		Log-Logistic	
	μ	σ	α	β
1991	0.0850	0.0812	2.1676	0.0579
1994	0.1712	0.1065	3.1489	0.1513
1997	0.2549	0.1452	3.9509	0.2669
2000	0.1984	0.1202	2.1468	0.1069
2003	0.1944	0.1538	1.7615	0.1051

Figure 3 shows graphically votes distribution of PRD in 2003.

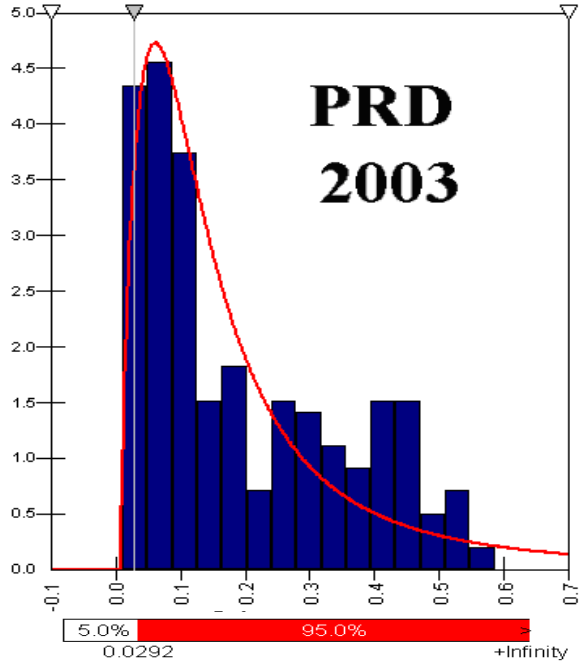

Figure 3: Votes Distribution Graphic of PRD in 2003 in the Electoral Districts of the Country.

2.1. Dynamics of the Political Parties in Mexico

Historically there is a tendency of political parties to disappear when the α parameter of the votes distribution of each party in the different federal entities, adjusted to the log-logistic distribution, is minor to 2. It is necessary to emphasize that is not a forceful fact to affirm the disappearance of some political party in later years. Nevertheless, tendencies and a greater possibility are observed to such event.

Analyzing the information of PAN and PRI, data shows to adjust with little probability to a heavy tails distribution behavior. It means that these parties do not show voters' absence in almost all of the federal entities. Contrary, the coalition with PT, PAS, Convergencia and PSN was not favorable for PRD in 2000 since its α parameter indicated a decreasing tendency. PRD gave signs of the disappearance of its sympathizers on the federal entities (Guzmán 2004), placing below the $\alpha = 2$ limit in 2003. But, the Working Party (PT by its abbreviation in Spanish) was favored by the coalition since in 2003 had an alpha parameter equal to 2.75 which indicates, unlike last years, a growth of its sympathizers, being outlined for later years as a national party. The Mexican Ecologist Party (PVEM by its abbreviation in Spanish) shows an α growth from 1991 to 2003 working in coalition with PAN in 2000 and with PRI in 2003, improving its electoral presence. The situation of minor parties such as PSN, PAS, MP, PLM was not favorable.

Table 4: Shape Parameter α of the Log-Logistic Distribution. Without Log-Logistic Distribution, didn't participate, participated in coalition

Parties	1991	1994	1997	2000	2003
PAN	2.55		8.15		
PRI					
PRD	2.16	3.14	3.95	**2.28**	1.76
PT	1.52	2.19	1.88		2.75
PVEM	1.81	2.28	1.94		**3.65**
Converge					1.37
PSN					1.62
PAS					1.70
MP					2.77
PLM					1.70
Fuerza C					3.43
PCD				2.26	
PARM	2.21	2.11		3.00	
DSPPN				2.06	
PC			2.24		
PPS	2.55	3.59	2.28		
PDM	1.81		1.73		
PFCRN	2.38	2.38			
UNO_PDM		**1.84**			
PRT	1.27				

3. RESULTS OF THE DYNAMIC PROCESS

Based on previous data, it was developed a computational program in Netlogo 4.0.2 (figure 4) that

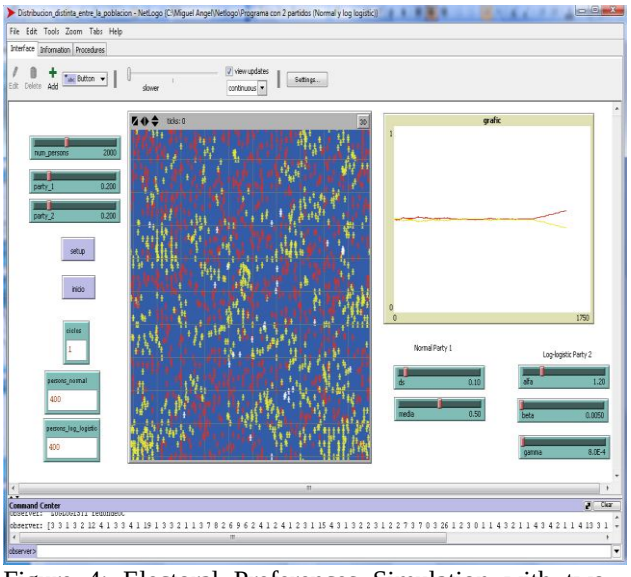

Figure 4: Electoral Preferences Simulation with two Parties with Netlogo 4.0.2 Software

simulates the statistical behavior of two parties with the same amount of initial voters but with different distribution of its voters over the region. Two types of distributions were considered: the normal and the log-logistic distribution.

Table 5 shows the results of 30 simulations of the people with preference towards some party with normal distribution and another with log-logistic distribution. A value of the α parameter variable from 1.0 to 2.0, a normal distribution with constant parameters, as well as the number of people constant (4,000) and the same number of initial people of both parties (400) with a defined preference towards a party.

Table 5: Results of 30 Simulations of People with Preference towards some Party

Alfa value	Number of times that won a party with	
	Normal distribution	Log-logistic distribution
1	30	0
1.1	28	2
1.2	27	3
1.3	29	1
1.4	26	4
1.5	26	4
1.6	24	6
1.7	22	8
1.8	25	5
1.9	20	10
2	20	10
3	18	12
4	16	14

Table 5 suggests that with an α<1.9 value, parties tend to lose and consequently to diminish their supporters over the years.

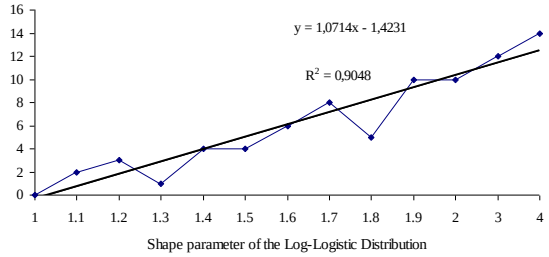

Figure 5: Number of times that won a Party with Log-Logistic Distribution

Then it was analyzed supporters final distribution type by each party, which resulted without changes for the normal distribution and changed the log-logistic for an exponential distribution (equation 3).

$$f(x) = \frac{e^{-x/\beta}}{\beta} \qquad (3)$$

Where "beta" is the mean.

The consequence is that the exponential distribution falls more quickly than the log-logistic distribution, suggesting a more quickly disappearance of the party.

3. CONCLUSIONS

The statistical distributions of the electoral presences of each political party are different, having the major parties a normal distribution of its voters at national level. On the other hand, the Democratic Revolution Party and the minor parties have a heavy tails distribution (log-logistic) which indicates a great concentration of voters in a few federal entities.

It was observed that when the log-logistic α parameter is minor to 2, political parties tend to disappear, although this is not an absolute truth. Nevertheless, this fact shows tendencies to debilitate the parties, which brings out in many cases the disappearance of them. This phenomenon was analyzed in an experimental way by developing a dynamic program (López 2000; Ross 2007; Stauffer 2003) where two parties interact (one with normal distribution and the other with log-logistic distribution). As a result we obtained a better participation of the party with log-logistic distribution while α was increased.

As part of a future work, the nonparticipation can be considered like another variable in the model. It is tried to complement the Mexican voters' pseudo-fractal network (Martínez and Balankin 2007) and improve the dynamic model for probabilistic tendencies of future electoral results. Political parties' behavior theories are still increasing. This creativity is commendable. Lamentably, the application remains limited.

It is pretended that the models focused on social system dynamics would be an important part in the decision making of the respective competitors, in order to fit its strategies, canalize its resources in a better way and to promote the vote between its supporters. For that motive it is necessary to improve the analysis, modeling and prediction of the electoral dynamic systems. In addition, these models could be useful to the electoral authorities to plan strategies to prevent the nonparticipation and to detect possible electoral frauds.

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Idalia Flores de la Mota studied a PhD in Operations Research in the Engineering Faculty of the National Autonomous University of Mexico (UNAM). She obtained an honorary mention for her master thesis and worked with Hamdy A. Taha for her doctorate thesis. She has attended to several national and international congresses and has been a member of Mexico's National Researchers System. Since 1990 is a full time professor at UNAM, teaching applied mathematics, integer programming, network programming and simulation. She worked as the Operations Research Section Chief for 10 years at UNAM'S Engineering Faculty. Her research interests are in networks simulation and logistic processes simulation.

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